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Sensitivity Analysis for Utility Maximization: A Study on Lagrange Multipliers and Commodity Coupons

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Abstract: *In this study sensitivity analysis among Lagrange multipliers and commodity coupons is discussed with detailed mathematical analysis. In economics, the utility maximization method is essential for increasing sustainable production and profit maximization. Two types of constraints, such as budget constraint, and coupon constraint are used to operate sensitivity analysis; consequently, two Lagrange multipliers are applied for the detailed mathematical analysis.*

Keywords: Lagrange multipliers, commodity coupon, sensitivity analysis, utility maximization

JEL Codes: C02, C51, C53, C61, C67, C68, D41, D62, E17, F63

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1. Introduction

Mathematical modeling in economics is the application of mathematics in economics that operates among prices, production, employment, saving, investment, etc. [Samuelson, 1947]. At present, it successfully covers many leading fields of modern society, such as economics, sociology, psychology, political science, etc. [Zheng & Liu, 2022]. The utility is an important and fundamental concept in modern economics [Fishburn, 1970]. It is the property in any object that produces benefit, advantage, pleasure, good, or happiness (positive utility), i.e., to prevent the happening of mischief, pain, evil, or unhappiness (negative utility) of a particular individual or a community of the society [Bentham, 1780; Castro & Araujo, 2019]. In every society, individuals want to obtain the highest level of satisfaction by the use of their purchasing goods and utility maximization helps in this favor [Kirsh, 2017]. Utility helps organizations to achieve maximum profit within the fixed budget and available technologies, and efficient use of raw materials [Eaton & Lipsey, 1975]. The notion of utility was established in the late 18th century by the English moral philosopher, jurist, and social reformer Jeremy Bentham (1748-1832) and English philosopher, political economist, Member of Parliament (MP), and civil servant John Stuart Mill (1806-1873) [Bentham, 1780; Marshall, 1920].

In this study, we have tried to discuss sensitivity analysis among Lagrange multipliers and commodity coupons during utility maximization, where we have used two constraints; budget constraint and coupon constraint. In the mathematical modeling, we have worked with the determinant of bordered 6×6 Hessian matrix and 6×10 Jacobian matrix, and have used implicit function theorem. In the analysis, we have tried our best to achieve the best result through the application of the scientific method of optimization.

2. Literature Review

In any research area, the literature review section is an introductory section of research, where the seminal works of previous researchers in the same field within the existing knowledge are highlighted [Polit & Hungler, 2013]. In 1928, two world-famous American scholars; mathematician Charles W. Cobb (1875-1949) and economist Paul H. Douglas (1892-1976), in a seminal paper provided the mathematical procedures of production functions; which is known as Cobb-Douglas production function [Cobb & Douglas, 1928]. Later in 1984, another two American researchers; mathematician John V. Baxley and economist John C. Moorhouse, worked on production functions, and they also provided the utility maximization policy [Baxley & Moorhouse, 1984]. David Gauthier believes that a rational and moral economic person seeks to maximize utility [Gauthier, 1975]. F. Thomas Juster gives a brief history of the development of utility theory. He examines the role of goods and services that produce utility [Juster, 1990].

Famous Bangladeshi mathematician and physicist Jamal Nazrul Islam (1939-2013) and his coauthors in two different papers have discussed both profit maximization and utility maximization [Islam et al., 2010a,b]. Young and novel researcher Lia Roy and her coauthors have provided a series of appendices to calculate cost minimization by the use of the Cobb-Douglas production function [Roy et al., 2021). Pahlaj Moolio and his coauthors have worked to form economic models with the Lagrange multiplier [Moolio et al., 2009]. Devajit Mohajan and Haradhan Kumar Mohajan have discussed utility maximization and profit maximization policies. They have studied the Cobb-Douglas production function with detailed mathematical analysis [Mohajan et al., 2013; Mohajan, 2022, Mohajan & Mohajan, 2022a].

3. Methodology of the Study

The academicians take the research as an essential and influential work to lead an academic empire [Pandey & Pandey, 2015]. The methodology is a guideline to perform good research that helps the researchers to increase the trust of a reader in the research findings [Kothari, 2008]. Hence, research methodology is a way for the researchers for organizing, planning, designing, and conducting good research [Legesse, 2014]. It helps novice researchers to identify research areas and projects within their capacities [Blessing et al., 1998].

In this study we have tried our best to maintain reliability and validity, and have tried to cite references properly in the text and reference list [Mohajan, 2017b]. We have used both published and unpublished secondary data sources of optimization to prepare the research paper [Mohajan, 2018b, 2020]. We have also taken help from journal articles, conference papers, published books, and handbooks, the internet, websites, etc.

4. Objective of the Study

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The prime objective of this article is to discuss sensitivity analysis among Lagrange multipliers and commodity coupons for utility maximization policy. In mathematical economics, the property of a commodity that enables it to satisfy human wants is considered a utility. The other related subordinate objectives are:

- to demonstrate the mathematical calculations elaborately,
- to show the economic predictions appropriately, and
- to adjust the economic properties with mathematics.

5. An Economic Model of Utility

We consider that there are only four commodities in society, such as E_1, E_2, E_3, and E_4. Let the consumers in the society willing to purchase only e_1, e_2, e_3, and e_4 amounts from these four commodities E_1, E_2, E_3, and E_4, respectively. The utility function for these four commodities can be written as [Islam et al., 2010a; Mohajan, 2018a; Mohajan & Mohajan, 2022b],

$$u(e_1, e_2, e_3, e_4) = e_1 e_2 e_3 e_4 . \tag{1}$$

Now we consider that the budget constraint of the consumers is [Mohajan, 2021a],

$$B(e_1, e_2, e_3, e_4) = p_1 e_1 + p_2 e_2 + p_3 e_3 + p_4 e_4 \tag{2}$$

where p_1, p_2, p_3, and p_4 are the prices of per unit of commodities e_1, e_2, e_3, and e_4, respectively. Now we consider that the coupon constraint is [Mohajan & Mohajan, 2022d],

$$N(e_1, e_2, e_3, e_4) = n_1 e_1 + n_2 e_2 + n_3 e_3 + n_4 e_4 , \tag{3}$$

where n_1, n_2, n_3, and n_4 are the coupons necessary to purchase a unit of commodity of e_1, e_2, e_3, and e_4, respectively.

Using (1), (2), and (3) we can present the Lagrangian function $U(e_1, e_2, e_3, e_4, \mu_1, \mu_2)$ as [Baxley & Moorhouse, 1984; Mohajan & Mohajan, 2022a],

$$\begin{aligned} U(e_1, e_2, e_3, e_4, \mu_1, \mu_2) &= e_1 e_2 e_3 e_4 + \mu_1 (B - p_1 e_1 - p_2 e_2 - p_3 e_3 - p_4 e_4) \\ &\quad + \mu_2 (N - n_1 e_1 - n_2 e_2 - n_3 e_3 - n_4 e_4). \end{aligned} \tag{4}$$

Lagrangian function (4) is a 6-dimensional unconstrained problem that maximizes utility functions; where μ_1 and μ_2 are two Lagrange multipliers.

Now taking first and second order, and also cross-partial derivatives in (4) we obtain [Islam et al. 2009a,b; Mohajan & Mohajan, 2022c];

$$\begin{aligned} &B_1 = p_1, \quad B_2 = p_2, \quad B_3 = p_3, \quad B_4 = p_4; \\ &N_1 = n_1, \quad N_2 = n_2, \quad N_3 = n_3, \quad N_4 = n_4; \end{aligned} \tag{5}$$

$$\begin{aligned} &U_{11} = 0, \quad U_{12} = U_{21} = e_3 e_4, \quad U_{13} = U_{31} = e_2 e_4, \\ &U_{14} = U_{41} = e_2 e_3, \quad U_{22} = 0, \quad U_{23} = U_{32} = e_1 e_4, \\ &U_{24} = U_{42} = e_1 e_3, \quad U_{33} = 0, \quad U_{34} = U_{43} = e_1 e_2, \quad U_{44} = 0, \end{aligned} \tag{6}$$

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where $\frac{\partial B}{\partial e_1}=B_1$, $\frac{\partial B}{\partial e_2}=B_2$, $\frac{\partial N}{\partial e_1}=N_1$, $\frac{\partial N}{\partial e_2}=N_2$, $\frac{\partial U}{\partial e_1}=U_1$, $\frac{\partial U}{\partial e_3}=U_3$, $\frac{\partial^2 U}{\partial e_2^2}=U_{22}$, etc. are partial differentiations of multivariate functions. Now we consider the determinant of bordered Hessian matrix [Mohajan, 2017a; Ferdous & Mohajan, 2022],

$$|H|=\begin{vmatrix} 0 & 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & 0 & -N_1 & -N_2 & -N_3 & -N_4 \\ -B_1 & -N_1 & U_{11} & U_{12} & U_{13} & U_{14} \\ -B_2 & -N_2 & U_{21} & U_{22} & U_{23} & U_{24} \\ -B_3 & -N_3 & U_{31} & U_{32} & U_{33} & U_{34} \\ -B_4 & -N_4 & U_{41} & U_{42} & U_{43} & U_{44} \end{vmatrix}. \quad (7)$$

For convenience we use $p_3=p_1$ and $p_4=p_2$, i.e., the amount of a pair of prices are the same, and also $n_3=n_1$ and $n_4=n_2$, i.e., a pair of coupon numbers are the same. Now we consider that in the expansion of (7) every term contains $p_1p_2n_1n_2$, then for utility maximization, from (7) we can derive [Mohajan & Mohajan, 2022e];

$$|H|=-2p_1p_2n_1n_2<0. \quad (8)$$

We can determine the Lagrange multiplier $\mu_1>0$ as [Mohajan & Mohajan, 2022b,d],

$$\mu_1=e_3e_4\frac{e_2n_2-e_1n_1}{n_2p_1-n_1p_2} \quad (9)$$

where $n_2p_1\neq n_1p_2$.

We can calculate the Lagrange multiplier $\mu_2>0$ as [Mohajan & Mohajan, 2022b,e],

$$\mu_2=e_3e_4\frac{e_2p_2-e_1p_1}{n_1p_2-n_2p_1} \quad (10)$$

where $n_1p_2\neq n_2p_1$.

For e_1, e_2, e_3, e_4, μ_1, and μ_2 in terms of p_1, p_2, p_3, p_4, n_1, n_2, n_3, n_4, *B*, and *N* we can calculate sixty partial derivatives, such as $\frac{\partial \mu_1}{\partial p_1}$, $\frac{\partial \mu_2}{\partial p_1}$, …, $\frac{\partial \mu_1}{\partial n_1}$, $\frac{\partial \mu_2}{\partial n_1}$, …, $\frac{\partial e_1}{\partial p_1}$, …, $\frac{\partial e_1}{\partial n_1}$, …, $\frac{\partial \mu_1}{\partial B}$, …, $\frac{\partial \mu_1}{\partial N}$, etc., [Islam et al., 2011; Mohajan, 2021c]. Now we consider 6×6 Hessian and Jacobian matrix as [Mohajan, 2021b; Mohajan & Mohajan, 2022a; Roy et al., 2021];

$$J=H=\begin{vmatrix} 0 & 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & 0 & -N_1 & -N_2 & -N_3 & -N_4 \\ -B_1 & -N_1 & U_{11} & U_{12} & U_{13} & U_{14} \\ -B_2 & -N_2 & U_{21} & U_{22} & U_{23} & U_{24} \\ -B_3 & -N_3 & U_{31} & U_{32} & U_{33} & U_{34} \\ -B_4 & -N_4 & U_{41} & U_{42} & U_{43} & U_{44} \end{vmatrix} \quad (11)$$

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which is non-singular at the optimum point $\left(e_1^*, e_2^*, e_3^*, e_4^*, \mu_1^*, \mu_2^*\right)$. Since the second-order conditions have been satisfied, so the determinant of (11) does not vanish at the optimum, i.e., $|J| = |H|$; and we apply the implicit function theorem. We have total 16 variables in our study, such as μ_1, μ_2, e_1, e_2, e_3, e_4 p_1, p_2, p_3, p_4, n_1, n_2, n_3, n_4, B, and N. By the implicit function theorem, we can write [Islam et al., 2010a; Mohajan et al., 2013];

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \mathbf{G}\left(p_1, p_2, p_3, p_4, n_1, n_2, n_3, n_4, B, N\right). \quad (12)$$

Now the 6×10 Jacobian matrix for $\mathbf{G}$; regarded as $J_G = \dfrac{\partial(\mu_1, \mu_2, e_1, e_2, e_3, e_4)}{\partial(p_1, p_2, p_3, p_4, n_1, n_2, n_3, n_4, B, N)}$, and is presented by [Mohajan, 2021a; Mohajan & Mohajan, 2022a],

$$J_G = \begin{bmatrix} \frac{\partial \mu_1}{\partial p_1} & \frac{\partial \mu_1}{\partial p_2} & \frac{\partial \mu_1}{\partial p_3} & \frac{\partial \mu_1}{\partial p_4} & \frac{\partial \mu_1}{\partial n_1} & \frac{\partial \mu_1}{\partial n_2} & \frac{\partial \mu_1}{\partial n_3} & \frac{\partial \mu_1}{\partial n_4} & \frac{\partial \mu_1}{\partial B} & \frac{\partial \mu_1}{\partial N} \\ \frac{\partial \mu_2}{\partial p_1} & \frac{\partial \mu_2}{\partial p_2} & \frac{\partial \mu_2}{\partial p_3} & \frac{\partial \mu_2}{\partial p_4} & \frac{\partial \mu_2}{\partial n_1} & \frac{\partial \mu_2}{\partial n_2} & \frac{\partial \mu_2}{\partial n_3} & \frac{\partial \mu_2}{\partial n_4} & \frac{\partial \mu_2}{\partial B} & \frac{\partial \mu_2}{\partial N} \\ \frac{\partial e_1}{\partial p_1} & \frac{\partial e_1}{\partial p_2} & \frac{\partial e_1}{\partial p_3} & \frac{\partial e_1}{\partial p_4} & \frac{\partial e_1}{\partial n_1} & \frac{\partial e_1}{\partial n_2} & \frac{\partial e_1}{\partial n_3} & \frac{\partial e_1}{\partial n_4} & \frac{\partial e_1}{\partial B} & \frac{\partial e_1}{\partial N} \\ \frac{\partial e_2}{\partial p_1} & \frac{\partial e_2}{\partial p_2} & \frac{\partial e_2}{\partial p_3} & \frac{\partial e_2}{\partial p_4} & \frac{\partial e_2}{\partial n_1} & \frac{\partial e_2}{\partial n_2} & \frac{\partial e_2}{\partial n_3} & \frac{\partial e_2}{\partial n_4} & \frac{\partial e_2}{\partial B} & \frac{\partial e_2}{\partial N} \\ \frac{\partial e_3}{\partial p_1} & \frac{\partial e_3}{\partial p_2} & \frac{\partial e_3}{\partial p_3} & \frac{\partial e_3}{\partial p_4} & \frac{\partial e_3}{\partial n_1} & \frac{\partial e_3}{\partial n_2} & \frac{\partial e_3}{\partial n_3} & \frac{\partial e_3}{\partial n_4} & \frac{\partial e_3}{\partial B} & \frac{\partial e_3}{\partial N} \\ \frac{\partial e_4}{\partial p_1} & \frac{\partial e_4}{\partial p_2} & \frac{\partial e_4}{\partial p_3} & \frac{\partial e_4}{\partial p_4} & \frac{\partial e_4}{\partial n_1} & \frac{\partial e_4}{\partial n_2} & \frac{\partial e_4}{\partial n_3} & \frac{\partial e_4}{\partial n_4} & \frac{\partial e_4}{\partial B} & \frac{\partial e_4}{\partial N} \end{bmatrix}. \quad (13)$$

$$= -J^{-1}\begin{bmatrix} -e_1 & -e_2 & -e_3 & -e_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -e_1 & -e_2 & -e_3 & -e_4 & 0 & 1 \\ -\mu_1 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu_1 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu_1 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & -\mu_2 & 0 & 0 \end{bmatrix}. \quad (14)$$

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The inverse of Jacobian matrix is, $J^{-1} = \frac{1}{|J|}C^T$, where $C = (C_{ij})$, the matrix of cofactors of J, where T indicates transpose, then (14) becomes [Mohajan, 2017a; Islam et al., 2009b, 2011],

$$J_G = -\frac{1}{|J|}C^T \begin{bmatrix} -e_1 & -e_2 & -e_3 & -e_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -e_1 & -e_2 & -e_3 & -e_4 & 0 & 1 \\ -\mu_1 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu_1 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu_1 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & -\mu_2 & 0 & 0 \end{bmatrix}. \quad (15)$$

Now 6×6 transpose matrix C^T can be represented by,

$$C^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} & C_{61} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} & C_{62} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} & C_{63} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} & C_{64} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{65} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix}. \quad (16)$$

Using (16) we can write (13) as a 6×10 Jacobian matrix [Mohajan & Mohajan, 2022b];

$$J_G = -\frac{1}{|J|}\begin{bmatrix} -e_1C_{11}-\mu_1C_{31} & -e_2C_{11}-\mu_1C_{41} & -e_3C_{11}-\mu_1C_{51} & -e_4C_{11}-\mu_1C_{61} & -e_1C_{21}-\mu_2C_{31} \\ -e_1C_{12}-\mu_1C_{32} & -e_2C_{12}-\mu_1C_{42} & -e_3C_{12}-\mu_1C_{52} & -e_4C_{12}-\mu_1C_{62} & -e_1C_{22}-\mu_2C_{32} \\ -e_1C_{13}-\mu_1C_{33} & -e_2C_{13}-\mu_1C_{43} & -e_3C_{13}-\mu_1C_{53} & -e_4C_{13}-\mu_1C_{63} & -e_1C_{23}-\mu_2C_{33} \\ -e_1C_{14}-\mu_1C_{34} & -e_2C_{14}-\mu_1C_{44} & -e_3C_{14}-\mu_1C_{54} & -e_4C_{14}-\mu_1C_{64} & -e_1C_{24}-\mu_2C_{34} \\ -e_1C_{15}-\mu_1C_{35} & -e_2C_{15}-\mu_1C_{45} & -e_3C_{15}-\mu_1C_{55} & -e_4C_{15}-\mu_1C_{65} & -e_1C_{25}-\mu_2C_{35} \\ -e_1C_{16}-\mu_1C_{36} & -e_2C_{16}-\mu_1C_{46} & -e_3C_{16}-\mu_1C_{56} & -e_4C_{16}-\mu_1C_{66} & -e_1C_{26}-\mu_2C_{36} \end{bmatrix.}$$

$$\begin{matrix} -e_2C_{21}-\mu_2C_{41} & -e_3C_{21}-\mu_2C_{51} & -e_4C_{21}-\mu_2C_{61} & C_{11} & C_{21} \\ -e_2C_{22}-\mu_2C_{42} & -e_3C_{22}-\mu_2C_{52} & -e_4C_{22}-\mu_2C_{62} & C_{12} & C_{22} \\ -e_2C_{23}-\mu_2C_{43} & -e_3C_{23}-\mu_2C_{53} & -e_4C_{23}-\mu_2C_{63} & C_{13} & C_{23} \\ -e_2C_{24}-\mu_2C_{44} & -e_3C_{24}-\mu_2C_{54} & -e_4C_{24}-\mu_2C_{64} & C_{14} & C_{24} \\ -e_2C_{25}-\mu_2C_{45} & -e_3C_{25}-\mu_2C_{55} & -e_4C_{25}-\mu_2C_{65} & C_{15} & C_{25} \\ -e_2C_{26}-\mu_2C_{46} & -e_3C_{26}-\mu_2C_{56} & -e_4C_{26}-\mu_2C_{66} & C_{16} & C_{26} \end{matrix}\Bigg]. \quad (17)$$

6. Sensitivity Analysis

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Now we analyze the nature of **the** Lagrange multiplier μ_1 when the quantity of surrendering coupons n_1 for the purchasing of the commodity e_1 increases. Taking T_{15}, (i.e., term of 1st row and 5th column) from both sides of (17) we get [Islam et al., 2011; Mohajan & Mohajan, 2022e],

$$\frac{\partial \mu_1}{\partial n_1} = \frac{e_1}{|J|}[C_{21}] + \frac{\mu_2}{|J|}[C_{31}]$$

$$= \frac{e_1}{|J|}\text{Cofactor of } C_{21} + \frac{\mu_2}{|J|}\text{Cofactor of } C_{31}$$

$$= -\frac{e_1}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ -N_1 & U_{11} & U_{12} & U_{13} & U_{14} \\ -N_2 & U_{21} & U_{22} & U_{23} & U_{24} \\ -N_3 & U_{31} & U_{32} & U_{33} & U_{34} \\ -N_4 & U_{41} & U_{42} & U_{43} & U_{44} \end{vmatrix} + \frac{\mu_2}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & -N_1 & -N_2 & -N_3 & -N_4 \\ -N_2 & U_{21} & U_{22} & U_{23} & U_{24} \\ -N_3 & U_{31} & U_{32} & U_{33} & U_{34} \\ -N_4 & U_{41} & U_{42} & U_{43} & U_{44} \end{vmatrix}$$

$$= -\frac{e_1}{|J|}\left\{ B_1\begin{vmatrix} -N_1 & U_{12} & U_{13} & U_{14} \\ -N_2 & U_{22} & U_{23} & U_{24} \\ -N_3 & U_{32} & U_{33} & U_{34} \\ -N_4 & U_{42} & U_{43} & U_{44} \end{vmatrix} - B_2\begin{vmatrix} -N_1 & U_{11} & U_{13} & U_{14} \\ -N_2 & U_{21} & U_{23} & U_{24} \\ -N_3 & U_{31} & U_{33} & U_{34} \\ -N_4 & U_{41} & U_{43} & U_{44} \end{vmatrix} \right.$$

$$\left. + B_3\begin{vmatrix} -N_1 & U_{11} & U_{12} & U_{14} \\ -N_2 & U_{21} & U_{22} & U_{24} \\ -N_3 & U_{31} & U_{32} & U_{34} \\ -N_4 & U_{41} & U_{42} & U_{44} \end{vmatrix} - B_4\begin{vmatrix} -N_1 & U_{11} & U_{12} & U_{13} \\ -N_2 & U_{21} & U_{22} & U_{23} \\ -N_3 & U_{31} & U_{32} & U_{33} \\ -N_4 & U_{41} & U_{42} & U_{43} \end{vmatrix} \right\} + \frac{\mu_2}{|J|}\left\{ -N_2\begin{vmatrix} -B_1 & -B_2 & -B_3 & -B_4 \\ -N_1 & -N_2 & -N_3 & -N_4 \\ U_{31} & U_{32} & U_{33} & U_{34} \\ U_{41} & U_{42} & U_{43} & U_{44} \end{vmatrix} \right.$$

$$\left. + N_3\begin{vmatrix} -B_1 & -B_2 & -B_3 & -B_4 \\ -N_1 & -N_2 & -N_3 & -N_4 \\ U_{21} & U_{22} & U_{23} & U_{24} \\ U_{41} & U_{42} & U_{43} & U_{44} \end{vmatrix} - N_4\begin{vmatrix} -B_1 & -B_2 & -B_3 & -B_4 \\ -N_1 & -N_2 & -N_3 & -N_4 \\ U_{21} & U_{22} & U_{23} & U_{24} \\ U_{31} & U_{32} & U_{33} & U_{34} \end{vmatrix} \right\}$$

$$= -\frac{e_1}{|J|}\left[B_1\left\{ -N_1\begin{vmatrix} U_{22} & U_{23} & U_{24} \\ U_{32} & U_{33} & U_{34} \\ U_{42} & U_{43} & U_{44} \end{vmatrix} - U_{12}\begin{vmatrix} -N_2 & U_{23} & U_{24} \\ -N_3 & U_{33} & U_{34} \\ -N_4 & U_{43} & U_{44} \end{vmatrix} + U_{13}\begin{vmatrix} -N_2 & U_{22} & U_{24} \\ -N_3 & U_{32} & U_{34} \\ -N_4 & U_{42} & U_{44} \end{vmatrix} - U_{14}\begin{vmatrix} -N_2 & U_{22} & U_{23} \\ -N_3 & U_{32} & U_{33} \\ -N_4 & U_{42} & U_{43} \end{vmatrix} \right\} \right.$$

$$- B_2\left\{ -N_1\begin{vmatrix} U_{21} & U_{23} & U_{24} \\ U_{31} & U_{33} & U_{34} \\ U_{41} & U_{43} & U_{44} \end{vmatrix} + U_{13}\begin{vmatrix} -N_2 & U_{21} & U_{24} \\ -N_3 & U_{31} & U_{34} \\ -N_4 & U_{41} & U_{44} \end{vmatrix} - U_{14}\begin{vmatrix} -N_2 & U_{21} & U_{23} \\ -N_3 & U_{31} & U_{33} \\ -N_4 & U_{41} & U_{43} \end{vmatrix} \right\} + B_3\left\{ -N_1\begin{vmatrix} U_{21} & U_{22} & U_{24} \\ U_{31} & U_{32} & U_{34} \\ U_{41} & U_{42} & U_{44} \end{vmatrix} \right.$$

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$$+U_{12}\begin{vmatrix}-N_2 & U_{21} & U_{24}\\ -N_3 & U_{31} & U_{34}\\ -N_4 & U_{41} & U_{44}\end{vmatrix} - U_{14}\begin{vmatrix}-N_2 & U_{21} & U_{22}\\ -N_3 & U_{31} & U_{32}\\ -N_4 & U_{41} & U_{42}\end{vmatrix}\Bigg\} - B_4\Bigg\{-N_1\begin{vmatrix}U_{21} & U_{22} & U_{23}\\ U_{31} & U_{32} & U_{33}\\ U_{41} & U_{42} & U_{43}\end{vmatrix} + U_{12}\begin{vmatrix}-N_2 & U_{21} & U_{23}\\ -N_3 & U_{31} & U_{33}\\ -N_4 & U_{41} & U_{43}\end{vmatrix}$$

$$-U_{13}\begin{vmatrix}-N_2 & U_{21} & U_{22}\\ -N_3 & U_{31} & U_{32}\\ -N_4 & U_{41} & U_{42}\end{vmatrix}\Bigg\}\Bigg] + \frac{\mu_2}{|J|}\Bigg[-N_2\Bigg\{-B_1\begin{vmatrix}-N_2 & -N_3 & -N_4\\ U_{32} & U_{33} & U_{34}\\ U_{42} & U_{43} & U_{44}\end{vmatrix} + B_2\begin{vmatrix}-N_1 & -N_3 & -N_4\\ U_{31} & U_{33} & U_{34}\\ U_{41} & U_{43} & U_{44}\end{vmatrix} - B_3\begin{vmatrix}-N_1 & -N_2 & -N_4\\ U_{31} & U_{32} & U_{34}\\ U_{41} & U_{42} & U_{44}\end{vmatrix}$$

$$+B_4\begin{vmatrix}-N_1 & -N_2 & -N_3\\ U_{31} & U_{32} & U_{33}\\ U_{41} & U_{42} & U_{43}\end{vmatrix}\Bigg\} + N_3\Bigg\{-B_1\begin{vmatrix}-N_2 & -N_3 & -N_4\\ U_{22} & U_{23} & U_{24}\\ U_{42} & U_{43} & U_{44}\end{vmatrix} + B_2\begin{vmatrix}-N_1 & -N_3 & -N_4\\ U_{21} & U_{23} & U_{24}\\ U_{41} & U_{43} & U_{44}\end{vmatrix} - B_3\begin{vmatrix}-N_1 & -N_2 & -N_4\\ U_{21} & U_{22} & U_{24}\\ U_{41} & U_{42} & U_{44}\end{vmatrix}$$

$$+B_4\begin{vmatrix}-N_1 & -N_2 & -N_3\\ U_{21} & U_{22} & U_{23}\\ U_{41} & U_{42} & U_{43}\end{vmatrix}\Bigg\} - N_4\Bigg\{-B_1\begin{vmatrix}-N_2 & -N_3 & -N_4\\ U_{22} & U_{23} & U_{24}\\ U_{32} & U_{33} & U_{34}\end{vmatrix} + B_2\begin{vmatrix}-N_1 & -N_3 & -N_4\\ U_{21} & U_{23} & U_{24}\\ U_{31} & U_{33} & U_{34}\end{vmatrix} - B_3\begin{vmatrix}-N_1 & -N_2 & -N_4\\ U_{21} & U_{22} & U_{24}\\ U_{31} & U_{32} & U_{34}\end{vmatrix}$$

$$+B_4\begin{vmatrix}-N_1 & -N_2 & -N_3\\ U_{21} & U_{22} & U_{23}\\ U_{31} & U_{32} & U_{33}\end{vmatrix}\Bigg\}\Bigg]$$

$$= -\frac{e_1}{|J|}\{-B_1N_1U_{23}U_{24}U_{34} - B_1N_1U_{23}U_{24}U_{34} - B_1N_2U_{12}U_{34}^2 + B_1N_4U_{12}U_{23}U_{34} + B_1N_3U_{12}U_{24}U_{34}$$

$$+B_1N_2U_{13}U_{24}U_{34} - B_1N_3U_{13}U_{24}^2 + B_1N_4U_{13}U_{23}U_{24} + B_1N_2U_{14}U_{23}U_{34} - B_1N_3U_{14}U_{23}U_{24} - B_1N_4U_{14}U_{23}^2$$

$$-B_2N_1U_{12}U_{14}U_{34} + B_2N_1U_{14}U_{23}U_{34} + B_2N_1U_{13}U_{24}U_{34} - B_2N_2U_{13}U_{14}U_{34} + B_2N_4U_{12}U_{13}U_{34}$$

$$+B_2N_3U_{13}U_{14}U_{24} - B_2N_4U_{24}U_{13}^2 - B_2N_2U_{13}U_{14}U_{34} + B_2N_3U_{12}U_{14}U_{34} - B_2N_3U_{23}U_{14}^2 + B_2N_4U_{13}U_{14}U_{23}$$

$$+B_3N_1U_{12}U_{24}U_{34} - B_3N_1U_{13}U_{24}^2 + B_3N_1U_{14}U_{23}U_{24} + B_3N_2U_{12}U_{14}U_{34} - K_3N_4U_{34}U_{12}^2 - B_3N_3U_{12}U_{14}U_{24}$$

$$+B_3N_4U_{12}U_{14}U_{23} + B_4N_1U_{12}U_{23}U_{34} + B_4N_1U_{13}U_{23}U_{24} - B_4N_1U_{14}U_{23}^2 + B_4N_2U_{12}U_{13}U_{34} - B_4N_3U_{34}U_{12}^2$$

$$+B_4N_3U_{12}U_{14}U_{23} - B_4N_4U_{12}U_{13}U_{23} - B_4N_2U_{24}U_{13}^2 + B_4N_2U_{13}U_{14}U_{23} + B_4N_3U_{12}U_{13}U_{24}$$

$$-B_4N_4U_{12}U_{13}U_{23}\} + \frac{\mu_2}{|J|}\{+B_1N_2^2U_{34}^2 - B_1N_2N_3U_{24}U_{34} - B_1N_2N_4U_{23}U_{34} - B_2N_1N_2U_{34}^2 + B_2N_2N_3U_{14}U_{34}$$

$$+B_2N_2N_4U_{13}U_{34} + B_3N_1N_2U_{24}U_{34} - B_3N_2^2U_{14}U_{34} - B_3N_2N_4U_{13}U_{24} + B_3N_2N_4U_{14}U_{23} + B_4N_1N_2U_{23}U_{34}$$

$$-B_4N_2^2U_{13}U_{34} + B_4N_2N_3U_{13}U_{24} - B_4N_2N_3U_{14}U_{23} - B_1N_2N_3U_{24}U_{34} + B_1N_3^2U_{24}^2 - B_1N_3N_4U_{23}U_{24}$$

$$-B_2N_1N_3U_{24}U_{34} - B_2N_3^2U_{14}U_{24} - B_2N_3N_4U_{12}U_{34} + B_2N_3N_4U_{14}U_{23} - B_3N_1N_3U_{24}^2 + B_3N_2N_3U_{14}U_{24}$$

$$+B_3N_3N_4U_{12}U_{24} + B_4N_1N_3U_{23}U_{24} + B_4N_2N_3U_{12}U_{34} - B_4N_2N_3U_{14}U_{23} - B_4N_3^2U_{12}U_{24} - B_1N_2N_4U_{23}U_{34}$$

$$-B_1N_3N_4U_{23}U_{24} + B_1N_4^2U_{23}^2 + B_2N_1N_4U_{23}U_{34} - B_2N_3N_4U_{12}U_{34} + B_2N_3N_4U_{13}U_{24} - B_2N_4^2U_{13}U_{23}$$

$$+B_3N_1N_4U_{23}U_{24}+B_3N_2N_4U_{12}U_{34}-B_3N_2N_4U_{13}U_{24}-B_3N_4^2U_{12}U_{23}-B_4N_1N_4U_{23}^2+B_4N_2N_4U_{13}U_{23}$$
$$+B_4N_3N_4U_{12}U_{23}\}$$
$$=-\frac{e_1}{|J|}\{-2p_1n_1e_1^3e_2e_3e_4-p_1n_2e_1^2e_2^2e_3e_4+p_1n_4e_1^2e_2e_3e_4^2+p_1n_3e_1^2e_2e_3^2e_4-p_1n_2e_1^2e_2^2e_3e_4-p_1n_3e_1^2e_2e_3^2e_4$$
$$+p_1n_4e_1^2e_2e_3e_4^2+p_1n_2e_1^2e_2^2e_3e_4-p_1n_3e_1^2e_2e_3^2e_4-p_1n_4e_1^2e_2e_3e_4^2-p_2n_1e_1e_2^2e_3^2e_4+p_2n_1e_1^2e_2^2e_3e_4$$
$$+p_2n_1e_1^2e_2^2e_3e_4-p_2n_2e_1e_2^3e_3e_4+p_2n_4e_1e_2^2e_3e_4^2+p_2n_3e_1e_2^2e_3^2e_4-p_2n_4e_1e_2^2e_3e_4^2-p_2n_2e_1e_2^3e_3e_4$$
$$+p_2n_3e_1e_2^2e_3^2e_4-p_2n_3e_1e_2^2e_3^2e_4+p_2n_4e_1e_2^2e_3e_4^2+p_3n_1e_1^2e_2e_3^2e_4-p_3n_1e_1^2e_2e_3^2e_4+p_3n_1e_1^2e_2e_3^2e_4$$
$$+p_3n_2e_1e_2^2e_3^2e_4-p_3n_4e_1e_2e_3^2e_4^2-p_3n_3e_1e_2e_3^3e_4+p_3n_4e_1e_2e_3^2e_4^2+p_4n_1e_1^2e_2e_3e_4^2+p_4n_1e_1^2e_2e_3e_4^2$$
$$-p_4n_1e_1^2e_2e_3e_4^2+p_4n_2e_1e_2^2e_3e_4^2-p_4n_3e_1e_2e_3^2e_4^2+p_4n_3e_1e_2e_3^2e_4^2-p_4n_4e_1e_2e_3e_4^3-p_4n_2e_1e_2^2e_3e_4^2$$
$$+p_4n_2e_1e_2^2e_3e_4^2+p_4n_3e_1e_2e_3^2e_4^2-p_4k_4e_1e_2e_3e_4^3\}+\frac{\mu_2}{|J|}\{p_1n_2^2e_1^2e_2^2-2p_1n_2n_3e_1^2e_2e_3-2p_1n_3n_4e_1^2e_3e_4$$
$$-2p_1n_2n_4e_1^2e_2e_4-p_1n_1n_2e_1^2e_2^2+p_2n_2n_3e_1e_2^2e_3+p_2n_2n_4e_1e_2^2e_4+p_3n_1n_2e_1^2e_2e_3-p_3n_2^2e_1e_2^2e_3$$
$$-2p_3n_2n_4e_1e_2e_3e_4+p_3n_2n_4e_1e_2e_3e_4+p_3n_2n_4e_1e_2e_3e_4+p_4n_2n_3e_1e_2e_3e_4-p_4n_2n_3e_1e_2e_3e_4+p_1n_3^2e_1^2e_3^2$$
$$-p_2n_1n_3e_1^2e_2e_3-p_2n_3^2e_1e_2e_3^2-2p_2n_3n_4e_1e_2e_3e_4+p_2n_3n_4e_1e_2e_3e_4-p_3n_1n_3e_1^2e_3^2+p_3n_3n_4e_1e_3^2e_4$$
$$+p_4n_1n_3e_1^2e_3e_4+p_4n_2n_3e_1e_2e_3e_4-p_4n_3^2e_1e_3^2e_4+p_1n_4^2e_1^2e_4^2+p_2n_1n_4e_1^2e_2e_4+p_2n_3n_4e_1e_2e_3e_4-p_2n_4^2e_1e_2e_4^2$$
$$+p_3n_1n_4e_1^2e_3e_4-p_3n_4^2e_1e_3e_4^2-p_4n_1n_4e_1^2e_4^2+p_4n_2n_4e_1e_2e_4^2+p_4n_3n_4e_1e_3e_4^2\}.$$
$$\frac{\partial\mu_1}{\partial n_1}=-\frac{e_1}{|J|}\{-2p_1n_1e_1^3e_2e_3e_4-p_1n_2e_1^2e_2^2e_3e_4+p_1n_4e_1^2e_2e_3e_4^2-p_1n_3e_1^2e_2e_3^2e_4+p_2n_1e_1^2e_2^2e_3e_4$$
$$-2p_2n_2e_1e_2^3e_3e_4+p_2n_3e_1e_2^2e_3^2e_4+p_2n_4e_1e_2^2e_3e_4^2+p_3n_1e_1^2e_2e_3^2e_4+p_3n_2e_1e_2^2e_3^2e_4-p_3n_3e_1e_2e_3^3e_4$$
$$+p_4n_1e_1^2e_2e_3e_4^2+p_4n_2e_1e_2^2e_3e_4^2+p_4n_3e_1e_2e_3^2e_4^2-2p_4n_4e_1e_2e_3e_4^3\}+\frac{\mu_2}{|J|}\{p_1n_2^2e_1^2e_2^2-2p_1n_2n_3e_1^2e_2e_3$$
$$-2p_1n_3n_4e_1^2e_3e_4+p_1n_4^2e_1^2e_4^2-2p_1n_2n_4e_1^2e_2e_4+p_1n_3^2e_1^2e_3^2-p_1n_1n_2e_1^2e_2^2+p_2n_2n_3e_1e_2^2e_3-p_2n_1n_3e_1^2e_2e_3$$
$$-p_2n_3^2e_1e_2e_3^2+p_2n_2n_4e_1e_2^2e_4+p_2n_1n_4e_1^2e_2e_4-p_2n_4^2e_1e_2e_4^2+p_3n_1n_2e_1^2e_2e_3-p_3n_2^2e_1e_2^2e_3-p_3n_1n_3e_1^2e_3^2$$
$$+p_3n_3n_4e_1e_3^2e_4+p_3n_1n_4e_1^2e_3e_4-p_3n_4^2e_1e_3e_4^2+p_4n_1n_3e_1^2e_3e_4+p_4n_2n_3e_1e_2e_3e_4-p_4n_3^2e_1e_3^2e_4-p_4n_1n_4e_1^2e_4^2$$
$$+p_4n_2n_4e_1e_2e_4^2+p_4n_3n_4e_1e_3e_4^2\}. \tag{18}$$

Using $n_3=n_1$ and $n_4=n_2$; and $p_3=p_1$ and $p_4=p_2$, and also $e_3=e_1$ and $e_2=e_4$ in (18) we get,

$$\frac{\partial\mu_1}{\partial n_1}=-\frac{e_1}{|J|}\{p_2n_1e_1^4e_2^2-3p_1n_1e_1^4e_2^2+3p_2n_1e_1^3e_2^3+p_2n_2e_1^3e_2^3+p_1n_2e_1^3e_2^3-3p_2n_2e_1^2e_2^4\}-\frac{\mu_2}{|J|}\{p_1n_1n_2e_1^3e_2$$
$$-2p_1n_2^2e_1^2e_2^2-p_1n_1n_2e_1^2e_2^2-2p_2n_1^2e_1^3e_2+4p_2n_1n_2e_1^2e_2^2\}. \tag{19}$$

Using $e_1=e_2=e$ in (10) we get,

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$$\mu_2 = \frac{e^3 (p_2 - p_1)}{n_1 p_2 - n_2 p_1} \tag{20}$$

where $n_1 p_2 \neq n_2 p_1$.

Using $e_1 = e_2 = e$ in (19) we get,

$$\frac{\partial \mu_1}{\partial n_1} = -\frac{e^7}{|J|}(4p_2 n_1 - 3p_1 n_1 + p_1 n_2 - 2p_2 n_2) - \frac{\mu_2 e^4}{|J|}\left(2p_1 n_2^2 + 2p_1 n_1 n_2 + 2p_2 n_1^2 - 4p_2 n_1 n_2\right). \tag{21}$$

Now using $|J| = |H| = -2p_1 p_2 n_1 n_2$ from (8) and the value of μ_2 from (20) in (21) we get,

$$\frac{\partial \mu_1}{\partial n_1} = -\frac{e^7}{2p_1 p_2 n_1 n_2}(3p_1 n_1 - 4p_2 n_1 - p_1 n_2 + 2p_2 n_2) + \frac{e^7 (p_2 - p_1)}{2p_1 p_2 n_1 n_2 (n_1 p_2 - n_2 p_1)}\left(2p_1 n_2^2 - 4p_2 n_1 n_2 + 2p_1 n_1 n_2 + 2p_2 n_1^2\right)$$

$$\frac{\partial \mu_1}{\partial n_1} = \frac{e^7\left(4p_1 p_2 n_2^2 - 5p_1 p_2 n_1^2 + 3p_1 p_2 n_1 n_2 + 6p_2^2 n_1^2 - 3p_1^2 n_2^2 + p_1^2 n_1 n_2 - 6p_2^2 n_1 n_2\right)}{2p_1 p_2 n_1 n_2 (n_1 p_2 - n_2 p_1)} \tag{22}$$

where $n_1 p_2 \neq n_2 p_1$ and $p_1 p_2 n_1 n_2 > 0$.

Now putting $n_1 = n_2 = n$ in (22) we get,

$$\frac{\partial \mu_1}{\partial n_1} = \frac{e^7}{p_2 n} > 0. \tag{23}$$

From inequality (23) we see that if the quantity of submitting coupons n_1 for the purchasing of the commodity e_1 increases, the level of marginal utility also increases. It seems that commodity e_1 is a superior good and it has no supplementary goods.

Now we analyze the nature of **the** Lagrange multiplier μ_2 when the quantity of submitting coupons n_1 for the purchasing of the commodity e_1 increases. Taking T_{25}, (i.e., term of 2nd row and 5th column) from both sides of (16) we get [Islam et al., 2010a; Mohajan, 2018a],

$$\frac{\partial \mu_2}{\partial n_1} = \frac{e_1}{|J|}[C_{22}] + \frac{\mu_2}{|J|}[C_{32}]$$

$$= \frac{e_1}{|J|}\text{Cofactor of } C_{22} + \frac{\mu_2}{|J|}\text{Cofactor of } C_{32}$$

$$= -\frac{e_1}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ -B_1 & U_{11} & U_{12} & U_{13} & U_{14} \\ -B_2 & U_{21} & U_{22} & U_{23} & U_{24} \\ -B_3 & U_{31} & U_{32} & U_{33} & U_{34} \\ -B_4 & U_{41} & U_{42} & U_{43} & U_{44} \end{vmatrix} - \frac{\mu_2}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & -N_1 & -N_2 & -N_3 & -N_4 \\ -B_2 & U_{21} & U_{22} & U_{23} & U_{24} \\ -B_3 & U_{31} & U_{32} & U_{33} & U_{34} \\ -B_4 & U_{41} & U_{42} & U_{43} & U_{44} \end{vmatrix}$$

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$$=\frac{e_1}{|J|}\left\{B_1\begin{vmatrix}-B_1 & U_{12} & U_{13} & U_{14}\\ -B_2 & U_{22} & U_{23} & U_{24}\\ -B_3 & U_{32} & U_{33} & U_{34}\\ -B_4 & U_{42} & U_{43} & U_{44}\end{vmatrix}-B_2\begin{vmatrix}-B_1 & U_{11} & U_{13} & U_{14}\\ -B_2 & U_{21} & U_{23} & U_{24}\\ -B_3 & U_{31} & U_{33} & U_{34}\\ -B_4 & U_{41} & U_{43} & U_{44}\end{vmatrix}+B_3\begin{vmatrix}-B_1 & U_{11} & U_{12} & U_{14}\\ -B_2 & U_{21} & U_{22} & U_{24}\\ -B_3 & U_{31} & U_{32} & U_{34}\\ -B_4 & U_{41} & U_{42} & U_{44}\end{vmatrix}\right.$$

$$\left.-B_4\begin{vmatrix}-B_1 & U_{11} & U_{12} & U_{13}\\ -B_2 & U_{21} & U_{22} & U_{23}\\ -B_3 & U_{31} & U_{32} & U_{33}\\ -B_4 & U_{41} & U_{42} & U_{43}\end{vmatrix}\right\}-\frac{\mu_2}{|J|}\left\{-B_2\begin{vmatrix}-B_1 & -B_2 & -B_3 & -B_4\\ -N_1 & -N_2 & -N_3 & -N_4\\ U_{31} & U_{32} & U_{33} & U_{34}\\ U_{41} & U_{42} & U_{43} & U_{44}\end{vmatrix}+B_3\begin{vmatrix}-B_1 & -B_2 & -B_3 & -B_4\\ -N_1 & -N_2 & -N_3 & -N_4\\ U_{21} & U_{22} & U_{23} & U_{24}\\ U_{41} & U_{42} & U_{43} & U_{44}\end{vmatrix}\right.$$

$$\left.-B_4\begin{vmatrix}-B_1 & -B_2 & -B_3 & -B_4\\ -N_1 & -N_2 & -N_3 & -N_4\\ U_{21} & U_{22} & U_{23} & U_{24}\\ U_{31} & U_{32} & U_{33} & U_{34}\end{vmatrix}\right\}$$

$$=\frac{e_1}{|J|}\left[B_1\left\{-B_1\begin{vmatrix}U_{22} & U_{23} & U_{24}\\ U_{32} & U_{33} & U_{34}\\ U_{42} & U_{43} & U_{44}\end{vmatrix}-U_{12}\begin{vmatrix}-B_2 & U_{23} & U_{24}\\ -B_3 & U_{33} & U_{34}\\ -B_4 & U_{43} & U_{44}\end{vmatrix}+U_{13}\begin{vmatrix}-B_2 & U_{22} & U_{24}\\ -B_3 & U_{32} & U_{34}\\ -B_4 & U_{42} & U_{44}\end{vmatrix}-U_{14}\begin{vmatrix}-B_2 & U_{22} & U_{23}\\ -B_3 & U_{32} & U_{33}\\ -B_4 & U_{42} & U_{43}\end{vmatrix}\right\}\right.$$

$$-B_2\left\{-B_1\begin{vmatrix}U_{21} & U_{23} & U_{24}\\ U_{31} & U_{33} & U_{34}\\ U_{41} & U_{43} & U_{44}\end{vmatrix}+U_{13}\begin{vmatrix}-B_2 & U_{21} & U_{24}\\ -B_3 & U_{31} & U_{34}\\ -B_4 & U_{41} & U_{44}\end{vmatrix}-U_{14}\begin{vmatrix}-B_2 & U_{21} & U_{23}\\ -B_3 & U_{31} & U_{33}\\ -B_4 & U_{41} & U_{43}\end{vmatrix}\right\}+B_3\left\{-B_1\begin{vmatrix}U_{21} & U_{22} & U_{24}\\ U_{31} & U_{32} & U_{34}\\ U_{41} & U_{42} & U_{44}\end{vmatrix}\right.$$

$$\left.+U_{12}\begin{vmatrix}-B_2 & U_{21} & U_{24}\\ -B_3 & U_{31} & U_{34}\\ -B_4 & U_{41} & U_{44}\end{vmatrix}-U_{14}\begin{vmatrix}-B_2 & U_{21} & U_{22}\\ -B_3 & U_{31} & U_{32}\\ -B_4 & U_{41} & U_{42}\end{vmatrix}\right\}-B_4\left\{-B_1\begin{vmatrix}U_{21} & U_{22} & U_{23}\\ U_{31} & U_{32} & U_{33}\\ U_{41} & U_{42} & U_{43}\end{vmatrix}+U_{12}\begin{vmatrix}-B_2 & U_{21} & U_{23}\\ -B_3 & U_{31} & U_{33}\\ -B_4 & U_{41} & U_{43}\end{vmatrix}\right.$$

$$\left.\left.-U_{13}\begin{vmatrix}-B_2 & U_{21} & U_{22}\\ -B_3 & U_{31} & U_{32}\\ -B_4 & U_{41} & U_{42}\end{vmatrix}\right\}\right]-\frac{\mu_2}{|J|}\left[-B_2\left\{-B_1\begin{vmatrix}-N_2 & -N_3 & -N_4\\ U_{32} & U_{33} & U_{34}\\ U_{42} & U_{43} & U_{44}\end{vmatrix}+B_2\begin{vmatrix}-N_1 & -N_3 & -N_4\\ U_{31} & U_{33} & U_{34}\\ U_{41} & U_{43} & U_{44}\end{vmatrix}-B_3\begin{vmatrix}-N_1 & -N_2 & -N_4\\ U_{31} & U_{32} & U_{34}\\ U_{41} & U_{42} & U_{44}\end{vmatrix}\right.\right.$$

$$\left.+B_4\begin{vmatrix}-N_1 & -N_2 & -N_3\\ U_{31} & U_{32} & U_{33}\\ U_{41} & U_{42} & U_{43}\end{vmatrix}\right\}+B_3\left\{-B_1\begin{vmatrix}-N_2 & -N_3 & -N_4\\ U_{22} & U_{23} & U_{24}\\ U_{42} & U_{43} & U_{44}\end{vmatrix}+B_2\begin{vmatrix}-N_1 & -N_3 & -N_4\\ U_{21} & U_{23} & U_{24}\\ U_{41} & U_{43} & U_{44}\end{vmatrix}-B_3\begin{vmatrix}-N_1 & -N_2 & -N_4\\ U_{21} & U_{22} & U_{24}\\ U_{41} & U_{42} & U_{44}\end{vmatrix}\right.$$

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$$+B_4\begin{vmatrix}-N_1 & -N_2 & -N_3\\ U_{21} & U_{22} & U_{23}\\ U_{41} & U_{42} & U_{43}\end{vmatrix}\Bigg\} - B_4\Bigg\{-B_1\begin{vmatrix}-N_2 & -N_3 & -N_4\\ U_{22} & U_{23} & U_{24}\\ U_{32} & U_{33} & U_{34}\end{vmatrix} + B_2\begin{vmatrix}-N_1 & -N_3 & -N_4\\ U_{21} & U_{23} & U_{24}\\ U_{31} & U_{33} & U_{34}\end{vmatrix} - B_3\begin{vmatrix}-N_1 & -N_2 & -N_4\\ U_{21} & U_{22} & U_{24}\\ U_{31} & U_{32} & U_{34}\end{vmatrix}$$

$$+B_4\begin{vmatrix}-N_1 & -N_2 & -N_3\\ U_{21} & U_{22} & U_{23}\\ U_{31} & U_{32} & U_{33}\end{vmatrix}\Bigg\}\Bigg]$$

$$=\frac{e_1}{|J|}\{-2B_1^2U_{23}U_{24}U_{34} - B_1B_2U_{12}U_{34}^2 + B_1B_4U_{12}U_{23}U_{34} + B_1B_3U_{12}U_{24}U_{34} + B_1B_2U_{13}U_{24}U_{34}$$
$$-B_1B_3U_{13}U_{24}^2 + B_1B_4U_{13}U_{23}U_{24} + B_1B_2U_{14}U_{23}U_{34} + B_1B_3U_{14}U_{23}U_{24} - B_1B_4U_{14}U_{23}^2 - B_1B_2U_{12}U_{34}^2$$
$$+B_1B_2U_{14}U_{23}U_{34} + B_1B_2U_{13}U_{24}U_{34} - B_2^2U_{13}U_{14}U_{34} - B_2B_4U_{12}U_{13}U_{34} + B_2B_3U_{13}U_{14}U_{24} - B_2B_4U_{13}^2U_{24}$$
$$-B_2^2U_{13}U_{14}U_{34} + B_2B_3U_{12}U_{14}U_{34} - B_2B_3U_{14}^2U_{23} + B_2B_4U_{13}U_{14}U_{23} - B_1B_3U_{12}U_{24}U_{34} - B_1B_3U_{13}U_{24}^2$$
$$+B_1B_3U_{14}U_{23}U_{24} + B_2B_3U_{12}U_{14}U_{34} - B_3B_4U_{12}^2U_{34} - B_3^2U_{12}U_{14}U_{24} + B_3B_4U_{12}U_{13}U_{24} + B_1B_4U_{12}U_{23}U_{34}$$
$$+B_1B_4U_{13}U_{23}U_{24} - B_1B_4U_{14}U_{23}^2 + B_2B_4U_{12}U_{13}U_{34} - B_3B_4U_{12}^2U_{34} + B_3B_4U_{12}U_{14}U_{23} - B_4^2V_{12}U_{13}U_{23}$$
$$-B_2B_4U_{13}^2U_{24} + B_2B_4U_{13}U_{14}U_{23} + B_3B_4U_{12}U_{13}U_{24} - B_4^2U_{12}U_{13}U_{23}\} - \frac{\mu_2}{|J|}\{B_1B_2N_2U_{34}^2 - B_1B_2N_3U_{24}U_{34}$$
$$-B_1B_2N_4U_{23}U_{34} - B_2^2N_1U_{34}^2 + B_2^2N_3U_{14}U_{34} - B_2^2N_4U_{13}U_{34} + B_2B_3N_1U_{24}U_{34} - B_2B_3N_2U_{14}U_{24}$$
$$-B_2B_3N_4U_{13}U_{24} + B_2B_3N_4U_{14}U_{23} + B_2B_4N_1U_{23}U_{34} - B_2B_4N_2U_{13}U_{34} + B_2B_4N_3U_{13}U_{24} - B_2B_4N_3U_{14}U_{23}$$
$$-B_1B_3N_2U_{24}U_{34} + B_1B_3N_3U_{24}^2 - B_1B_3N_4U_{23}U_{24} + B_2B_3N_1U_{24}U_{34} - B_2B_3N_2U_{14}U_{24} - B_2B_3N_4U_{12}U_{34}$$
$$+B_2B_3N_4U_{14}U_{23} - B_3^2N_1U_{24}^2 + B_3^2N_2U_{14}U_{24} + B_3^2N_4U_{12}U_{24} + B_3B_4N_1U_{23}U_{24} + B_3B_4N_2U_{12}U_{34}$$
$$-B_3B_4N_2U_{14}U_{23} - B_3B_4N_3U_{12}U_{24} - B_1B_4N_2U_{23}U_{34} - B_1B_4N_3U_{23}U_{24} + B_1B_4N_4U_{23}^2 + B_2B_4N_1U_{23}U_{34}$$
$$-B_2B_4N_3U_{12}U_{34} + B_2B_4N_3U_{13}U_{24} - B_2B_4N_4U_{13}U_{23} + B_3B_4N_1U_{23}U_{24} + B_3B_4N_2U_{12}U_{34} - B_3B_4N_2U_{13}U_{24}$$
$$-B_3B_4N_4U_{12}U_{23} - B_4^2N_1U_{23}^2 + B_4^2N_2U_{13}U_{23} + B_4^2N_3U_{12}U_{23}\}$$

$$\frac{\partial\mu_2}{\partial n_1} = \frac{e_1}{|J|}\{-2p_1^2e_1^3e_2e_3e_4 - p_1p_2e_1^2e_2^2e_3e_4 + p_1p_4e_1^2e_2e_3e_4^2 + p_1p_3e_1^2e_2e_3^2e_4 + p_1p_3e_1^2e_2^2e_3e_4 - p_1p_3e_1^2e_2e_3^2e_4$$
$$+p_1p_4e_1^2e_2e_3e_4^2 + p_1p_2e_1^2e_2^2e_3e_4 + p_1p_3e_1^2e_2e_3^2e_4 - p_1p_4e_1^2e_2e_3e_4^2 - p_1p_2e_1^2e_2^2e_3e_4 + 2p_1p_2e_1^2e_2^2e_3e_4$$
$$-p_2^2e_1e_2^3e_3e_4 - p_2p_4e_1e_2^2e_3e_4^2 + p_2p_3e_1e_2^2e_3^2e_4 - p_2p_4e_1e_2^2e_3e_4^2 - p_2^2e_1e_2^3e_3e_4 + p_2p_3e_1e_2^2e_3^2e_4 - p_2p_3e_1e_2^2e_3^2e_4$$
$$+p_2p_4e_1e_2^2e_3e_4^2 - p_1p_3e_1^2e_2e_3^2e_4 - p_1p_3e_1^2e_2e_3^2e_4 + p_1p_3e_1^2e_2e_3^2e_4 + p_2p_3e_1e_2^2e_3^2e_4 - p_3p_4e_1e_2e_3^2e_4^2$$
$$-p_3^2e_1e_2e_3^3e_4 + p_3p_4e_1e_2e_3^2e_4^2 + p_1p_4e_1^2e_2e_3e_4^2 + p_1p_4e_1^2e_2e_3e_4^2 - p_1p_4e_1^2e_2e_3e_4^2 + p_2p_4e_1e_2^2e_3e_4^2$$
$$-p_3p_4e_1e_2e_3^2e_4^2 + p_3p_4e_1e_2e_3^2e_4^2 - p_4^2e_1e_2e_3e_4^3 - p_2p_4e_1e_2^2e_3e_4^2 + p_2p_4e_1e_2^2e_3e_4^2 + p_3p_4e_1e_2e_3^2e_4^2$$
$$-p_4^2e_1e_2e_3e_4^3\} - \frac{\mu_2}{|J|}\{p_1p_2n_2e_1^2e_2^2 - p_1p_2n_3e_1^2e_2e_3 - p_1p_2n_4e_1^2e_2e_4 - p_2^2n_1e_1^2e_2^2 + p_2^2n_3e_1e_2^2e_3 - p_2^2n_4e_1e_2^2e_4$$

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$$+p_2p_3n_1e_1^2e_2e_3-p_2p_3n_2e_1e_2e_3^2-p_2p_3n_4e_1e_2e_3e_4+p_2p_3n_4e_1e_2e_3e_4+p_2p_4n_1e_1^2e_2e_4-p_2p_4n_2e_1e_2^2e_4$$
$$+p_2p_4n_3e_1e_2e_3e_4-p_2p_4n_3e_1e_2e_3e_4-p_2p_3n_2e_1^2e_2e_3-p_1p_3n_3e_1^2e_3^2-p_1p_3n_4e_1^2e_3e_4+p_2p_3n_1e_1^2e_2e_3$$
$$-p_2p_3n_2e_1e_2e_3^2-p_2p_3n_4e_1e_2e_3e_4+p_2p_3n_4e_1e_2e_3e_4-p_3^2n_1e_1^2e_3^2+p_3^2n_2e_1e_2e_3^2+p_3p_4n_1e_1^2e_3e_4$$
$$+p_3p_4n_2e_1e_2e_3e_4-p_3p_4n_2e_1e_2e_3e_4-p_3p_4n_3e_1e_3^2e_4-p_1p_4n_2e_1^2e_2e_4-p_1p_4n_3e_1^2e_3e_4+p_1p_4n_4e_1^2e_4^2$$
$$+p_2p_4n_1e_1^2e_2e_4-p_2p_4n_3e_1e_2e_3e_4+p_2p_4n_3e_1e_2e_3e_4-p_2p_4n_4e_1e_2e_4^2+p_3p_4n_1e_1^2e_3e_4+p_2p_4n_3e_1e_2e_3e_4$$
$$+p_3p_4n_2e_1e_2e_3e_4-p_3p_4n_2e_1e_2e_3e_4-p_3p_4n_4e_1e_3e_4^2-p_4^2n_1e_1^2e_4^2+p_4^2n_2e_1e_2e_4^2+p_4^2n_3e_1e_3e_4^2\}. \quad (24)$$

Using $n_3=n_1$ and $n_4=n_2$; and $p_3=p_1$ and $p_4=p_2$; and also $e_3=e_1$ and $e_4=e_2$ in (24) we get,

$$\frac{\partial\mu_2}{\partial n_1}=-\frac{e_1}{|J|}\{3p_1^2e_1^4e_2^2-7p_1p_2e_1^3e_2^3+4p_2^2e_1^2e_2^4\}+\frac{\mu_2}{|J|}\{2p_2^2n_2e_1e_2^3+2p_1p_2n_2e_1^2e_2^2-3p_2^2n_1e_1^2e_2^2+p_1p_2n_2e_1^3e_2$$
$$+3p_1^2n_1e_1^4-p_1^2n_2e_1^3e_2\}. \quad (25)$$

Using $e_1=e_2=e$ in (10) we get,

$$\mu_2=\frac{e^3(p_2-p_1)}{n_1p_2-n_2p_1} \quad (26)$$

where $n_1p_2\neq n_2p_1$.

Now using $|J|=|H|=-2p_1p_2n_1n_2$ from (8), $e_1=e_2=e$ and value of μ_2 from (26) in (25) we get,

$$\frac{\partial\mu_2}{\partial n_1}=\frac{e^7(3p_1^2-7p_1p_2+4p_2^2)}{2p_1p_2n_1n_2}-\frac{e^7(p_2-p_1)(3p_1p_2n_2+3p_1^2n_1-3p_2^2n_1-p_1^2n_2+2p_2^2n_2)}{2p_1p_2n_1n_2(n_1p_2-n_2p_1)}. \quad (27)$$

$$\frac{\partial\mu_2}{\partial n_1}=\frac{e^7(11p_1^2p_2n_2-10p_1p_2^2n_1-5p_1p_2^2n_2+3p_1^3n_1-4p_1^3n_1+7p_2^3n_1-p_2^3n_2)}{2p_1p_2n_1n_2(n_1p_2-n_2p_1)} \quad (28)$$

where $n_1p_2\neq n_2p_1$.

Now using $n_1=n_2=n$ in (28) we get,

$$\frac{\partial\mu_2}{\partial n_1}=\frac{e^7(2p_2-p_1)^2+p_2(p_2-6p_1)}{2p_1p_2n^2} \quad (29)$$

where $2p_1p_2n^2>0$.

Now if $p_2>6p_1$ in (29) we observe that,

$$\frac{\partial\mu_2}{\partial n_1}>0. \quad (30)$$

From the inequality (29) we have realized that if the quantity of submitting coupons n_1 for the purchasing of the commodity e_1 increases, the level of marginal utility also increases. Therefore, if the number of coupons n_1 increases one unit, marginal utility increases exactly μ_2 units. It seems that commodity e_1 is a superior good and it has no supplementary goods. In this situation, the organization should try to increase the production of the commodity e_1 depending on the local and global demand [Mohajan & Mohajan, 2022c,f].

Now if $p_2 < 1.9 p_1$ in (29) we observe that,

$$\frac{\partial \mu_2}{\partial n_1} < 0. \quad (31)$$

The inequality (31) indicates that if the quantity of submitting coupon n_1 for the purchasing of the commodity e_1 increases, the level of marginal utility decrease. That is, for the increase of coupons n_1, the marginal utility will decrease by exact μ_2 units. This situation seems reasonable result in the sense that a commodity e_1 has many substitutes, and hence consumers switch to substitutes when the price of a commodity e_1 goes up.

7. Conclusion

In this study, we have discussed sensitivity analysis among Lagrange multipliers and commodity coupons during utility maximization. Of course, the Lagrange multipliers method is a very useful and powerful analysis in multivariate calculus. We have realized that sensitivity analysis is very important for economists, and the producers can predict their future production. Throughout the study, we have tried to present the mathematical representations elaborately.

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